

## Final Report for ONR GRANT N00014-05-1-0166

Over the three years of 10/1/2004 through 9/30/2007, the grant supported the PI Éva Tardos, and provided partial support for students Zoya Svitkina, and Thanh Nguyen, Ara Hayrapetyan, Georgios Piliouras, and student and later postdoctoral fellow Tom Wexler. Tom left Cornell for a faculty position at Denison university, Ohio; Zoya is graduated this summer and left for a postdoc at Dartmouth; and Ara graduated this May and is now working at Credit Suisse in New York.

### Abstract

Many networks today are simultaneously built, operated, and used by multiple parties with a diverse set of goals and with constantly changing degree of cooperation and competition. The future of much of the complex technology developed today depends on our ability to successfully build and manage systems used by such diverse set of users, ensure that participants cooperate despite their diverse goals and interests. Examples include a wide range of networks military, though transportation networks, and Information networks like the Web and the Internet. Such large and decentralized networks provide amazing opportunities, but it also presents large challenges. The future of much of the complex technology developed today depends on our ability to successfully build and manage systems used by such diverse set of users, ensure that participants cooperate despite their diverse goals and interests.

One focus of the project was algorithmic game theory, and it aimed to understand what are simple and natural frameworks that lead to efficient systems for such heterogeneous user sets. We revisited some traditional algorithmic questions, such as network design, from the perspective of building networks for heterogeneous users. The main focus of the research was to approach some of the traditional algorithmic questions in networks from the perspective of game theory. We mostly focused on the quality of stable solutions obtained by selfish users, but also started to consider cooperative behavior of users.

A related focus was approximation algorithms. We considered approximation algorithms for some of the related optimization problems. We developed approximation algorithms for problems of influence maximization, graph partitioning problems, facility location with heterogeneous user sets, and disjoint path problems.

In addition to the new research reported below, I also spend time during the last three years writing a textbook on *Algorithm Design* (with Jon Kleinberg) that had now been adopted by a number of school as their textbook for the undergraduate or graduate algorithms course. This past year PI Éva Tardos have been editing a collected volume on *Algorithmic Game Theory* (with Nisan, Roughgarden, and Vazirani) that appeared this fall. I have co-authored three of the 29 chapters of the book. I wrote an introduction to game theory for computer scientist (with Vijay Vazirani), an introduction to “Price of anarchy” (with Tim Roughgarden), the area working to evaluate the quality of equilibria in games. Finally, I also wrote a survey on the current state of the art on network formation games (with Tom Wexler), and area that will be important both for understanding physical networks like the Internet, and also for understanding emerging social



networks the face-book. These networks are still in the developing stages, and our understanding can hopefully lead to design that will significantly enhance all of our lives.

- L. Blume, D. Easley, J. Kleinberg, and E. Tardos, Trading Networks with Price Setting Agents, in the ACM Conference on Electronic Commerce, 2007.  
<http://www.cs.cornell.edu/people/eva/traders.pdf>

In a wide range of markets, individual buyers and sellers often trade through intermediaries, who determine prices via strategic considerations. Typically, not all buyers and sellers have access to the same intermediaries, and they trade at correspondingly different prices that reflect their relative amounts of power in the market. We model this phenomenon using a game in which buyers, sellers, and traders engage in trade on a graph that represents the access each buyer and seller has to the traders. We show that the resulting game always has a subgame perfect Nash equilibrium, and that all equilibria lead to an efficient (i.e. socially optimal) allocation of goods, and we characterize how the profits obtained by the traders depend on the underlying graph.

- Ara Hayrapetyan, David Kempe, Martin Pál and Zoya Svitkina Unbalanced Graph Cuts European Symposium on Algorithms (ESA), 2005  
<http://www.cs.cornell.edu/~ara/papers/hkps05esa.pdf>

We introduce the Minimum-Size Bounded-Capacity Cut (MSBCC) problem, in which we are given a graph with an identified source and seek to find a cut minimizing the number of nodes on the source side subject to the constraint that the capacity may not exceed a prescribed bound. Besides being of interest in the study of graph cuts, this problem arises in many practical settings, such as epidemiology, disaster control, military containment, as well as finding dense subgraphs and communities in graphs. Our main result is a  $(1/\lambda, 1/(1 - \lambda))$  - bicriteria approximation algorithm for any  $0 \leq \lambda \leq 1$ , where the first parameter is the extent by which the desired capacity is violated, and the second parameter is the extent by which our solution is larger than the optimal solution that does not violate the capacity constraint.

- Ara Hayrapetyan, Éva Tardos and Tom Wexler. A network pricing game for selfish traffic, Distributed Computing (Special Issue PODC 05) Volume 19, Number 4, 2007, 255-266.  
<http://www.springerlink.com/content/x148746507861np7/?p=74c392d3d6fa495d9a92b0c778a44981&pi=19>

The Internet, unlike many small-scale networks, is built in a decentralized fashion and is controlled by a large number of disparate service providers who are not interested in any global optimization. Instead, the providers seek to maximize their own profit by charging users access to their service. Users themselves behave selfishly, optimizing over price and quality of service. Game theory provides a natural framework for a study of such a situation. However past work in this area has mostly focused on either the service providers or the network users, but not both. The paper introduces a new model for exploring the interaction of these two elements, in which network managers compete for users via prices

and quality of provided service. We study the extent to which competition between service providers hurts the overall social utility of the system.

- A. Hayrapetyan, E. Tardos, and T. Wexler. How Much Can Coalitions Hurt the Price of Anarchy. To appear in the Proceedings of the ACM Symposium on the Theory of Computing (STOC), 2006.

<http://www.cs.cornell.edu/~wexler/collusion-STOC06.ps>

We consider the question of how collusion alters the quality of solutions obtained in competitive games. The price of anarchy aims to measure the cost of the lack of coordination by comparing the quality of a Nash equilibrium to that of a centrally designed optimal solution. However, this notion assumes that players act not only selfishly, but also independently. If a subset of players collude, this can improve the social welfare of the participants, but it can also harm the welfare of those outside the coalition. The question we study is what is the effect of such collusion on the overall solution quality.

- D. Kempe, J. Kleinberg and E. Tardos, Influential Nodes in a Diffusion Model for Social Networks, to appear in the 32nd International Colloquium on Automata, Languages and Programming (ICALP) July, 2005.

We study the problem of how to spread an innovation in a network, such as a social network. We consider studies a general diffusion model termed "Decreasing Cascade Mode," that contains several standard models from the sociology literature. We prove that the greedy algorithm is a  $(1-1/e)$  approximation for the problem of how to spend a given marketing budget to reach as large a fraction of the network as possible in expectation.

- Jon Kleinberg and Éva Tardos: Algorithms Design, Addison-Wesley, 2005.

Undergraduate and graduate computer science textbook book on Algorithm Design that had now been adopted by a number of school as their textbook for the undergraduate or graduate algorithms course.

- Ephraim Korach, Thành Nguyen and Britta Peis. In the Proceedings of the 17th Annual ACM-SIAM Symposium on Discrete Algorithms, 2006.

<http://www.cam.cornell.edu/%7Eethanh/soda06.pdf>

König-Egerváry graphs (KEGs) are the graphs whose maximum size of a matching is equal to the minimum size of a vertex cover. We give an excluded subgraph characterization of KEGs by proving a more general result: excluded subgraph characterization of Red/Blue-split graphs. We show several consequences of this result including theorems of Demming-Sterboul, Lovsz, and Fides-Hammer. A refined result of Schrijver on the integral solution of certain systems of linear inequalities is also given.

- H. Lin, T. Roughgarden, E. Tardos, and A. Walkover. Braess's Paradox, Fibonacci Numbers, and Exponential Inapproximability, to appear in the 32nd International Colloquium on Automata, Languages and Programming (ICALP,05) July, 2005.



We give a worst case analysis of the severity of Braess paradox in multicommodity networks. We show that adding a single edge in a two-commodity network can lead to exponential increase in the delays of the resulting equilibrium routing.

- Thành Nguyen On the disjoint paths problem, Operation Research Letters, Volume 35, Issue 1, January 2007, pp. 10-16.  
<http://www.cam.cornell.edu/%7Ethan/disjoint.pdf>

We give a  $\sqrt{n}+1$  approximation algorithm for the edge disjoint paths problem in undirected graphs. This is currently the best known approximation guarantee. Our combinatorial technique also leads to  $O(\sqrt{n})$  approximation for directed acyclic graphs and directed graphs with edge capacity greater or equal to two.

- Thành Nguyen, Éva Tardos. Approximately Maximizing Efficiency and Revenue in Polyhedral Environments. ACM EC 2007.  
See also <http://www.cam.cornell.edu/thanh/paper/game.pdf>.

We consider a resource allocation game in convex environments. Convex environments model a wide range of problems, including bandwidth sharing, some models of Adword auctions and general resource allocation. We extend the fair sharing mechanism for such resource allocation games, and show that our mechanism simultaneously creates approximately efficient allocations and approximately maximizes revenue.

- Thành Nguyen, Éva Tardos. Parallel Imaging problem.  
<http://www.cam.cornell.edu/thanh/paper/mri.pdf>

We give a constant approximation for a problem motivated by Magnetic Resonance Imaging reconstruction. Our problem combines the metric labeling problem and linear algebra.

- Noam Nisan, Tim Roughgarden, E. Tardos, and Vijay Vazirani (eds) Algorithmic Game Theory, Cambridge University Press, 2007.  
[http://theory.stanford.edu/~tim/agt/toc\\_brief.html](http://theory.stanford.edu/~tim/agt/toc_brief.html)

Edited volume on algorithmic game theory, covering topics from computability of equilibria, mechanism design, quantifying the quality of equilibria, and a wide range of applications.

- T. Roughgarden and É. Tardos. Introduction to the Inefficiency of Equilibria, in *Algorithmic Game Theory*, Noam Nisan, Tim Roughgarden, Éva Tardos, Vijay Vazirani (eds), Cambridge University Press, 2007.  
<http://www.cs.cornell.edu/people/eva/agtchap17.pdf>

Instruction to quality of equilibria in games.

- Zoya Svitkina, and Eva Tardos. Facility Location with Hierarchical Facility Costs in the Proceedings of the 17th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA'06), 2006.

We study a generalization of the facility location problem in which the cost of an opened facility depends on the set of clients assigned to that facility, and this dependence can be

described by a certain type of function with hierarchical structure. We present a constant-factor approximation algorithm for this problem, based on the local search technique, which makes two kinds of local improvement moves: aggregate and disperse. We show that our proposed aggregate move can be extended to work for more general (not only hierarchical) facility cost functions.

- É. Tardos and V. Vazirani: Basic Solution Concepts and Computational Issues in Games. in *Algorithmic Game Theory*, Noam Nisan, Tim Roughgarden, Éva Tardos, Vijay Vazirani (eds), Cambridge University Press, 2007.

<http://www.cs.cornell.edu/people/eva/agtchap19.pdf>

Introduction to solution concepts and computational issues in games using applications and examples in computer science context.

- É. Tardos and T. Wexler: Network Formation Games, in *Algorithmic Game Theory*, Noam Nisan, Tim Roughgarden, Éva Tardos, Vijay Vazirani (eds), Cambridge University Press, 2007.

<http://www.cs.cornell.edu/people/eva/agtchap1.pdf>

A survey on network formation games including local connection games (where users are node of a graph and choose to form edges to some other nodes), global connection games (where users cooperate to build networks, but each have their own goals, and facility location games (where the players are the facilities and they attract customers via location and pricing).



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